1 Given that y is a prime number,

(b) express $\dfrac{3}{2-\sqrt{y}}$ in the form $\dfrac{a+b\sqrt{y}}{c-y}$ where a, b and c are integers.

$$\frac{3}{2-\sqrt{y}} \times \frac{(2+\sqrt{y})}{(2+\sqrt{y})} \text{ (1)} = \frac{3(2+\sqrt{y})}{(2-\sqrt{y})(2+\sqrt{y})}$$

$$= \frac{6+3\sqrt{y}}{4-y}$$

$\dfrac{6+3\sqrt{y}}{4-y}$ (1)

(2)

(Total for Question 1 is 2 marks)

2 (a) Show that $\left(6 + 2\sqrt{12}\right)^2 = 12\left(7 + 4\sqrt{3}\right)$

Show each stage of your working.

LHS : $(6+2\sqrt{12})(6+2\sqrt{12})$

$= 36 + 12\sqrt{12} + 12\sqrt{12} + 4(12)$ (1)

$= 36 + 24\sqrt{12} + 48$

$= 36 + 24\sqrt{4\times 3} + 48$

$= 36 + 24(2\sqrt{3}) + 48$ (1)

$= 36 + 48\sqrt{3} + 48$

$= 12(3 + 4\sqrt{3} + 4)$ (1)

$= 12(7 + 4\sqrt{3})$

(3)

(Total for Question 2 is 3 marks)

3 The area of a rectangle is 18 cm^2

The length of the rectangle is $(\sqrt{7} + 1)$ cm.

Without using a calculator and showing each stage of your working,

find the width of the rectangle.
Give your answer in the form $a\sqrt{b} + c$ where a, b and c are integers.

$$(\sqrt{7}+1) \times w = 18$$

$$w = \frac{18}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1} \quad (1)$$

$$= \frac{18\sqrt{7} - 18}{7-1} \quad (1)$$

$$= \frac{18\sqrt{7} - 18}{6}$$

$$w = 3\sqrt{7} - 3 \quad (1)$$

Area = length × width

$3\sqrt{7} - 3$ .... cm

(Total for Question 3 is 3 marks)

4 (b) Show that $\dfrac{2}{6-3\sqrt{2}}$ can be written in the form $\dfrac{a+\sqrt{a}}{b}$

where a and b are integers.
Show your working clearly.

$$\frac{2}{6-3\sqrt{2}} \times \frac{6+3\sqrt{2}}{6+3\sqrt{2}} \quad (1)$$ – eliminate surds from denominator

$$= \frac{2(6+3\sqrt{2})}{36-9(2)}$$

$$= \frac{12+6\sqrt{2}}{18} \quad (1)$$

$$= \frac{\cancel{6}(2+\sqrt{2})}{\cancel{6}(3)}$$

$$= \frac{2+\sqrt{2}}{3} \quad (1)$$ where $a = 2$, $b = 3$

(3)

(Total for Question 4 is 3 marks)

5 Given that $(8 - \sqrt{x})(5 + \sqrt{x}) = y\sqrt{x} + 21$ where x is a prime number and y is an integer,

find the value of x and the value of y.
Show each stage of your working clearly.

$(8 - \sqrt{x})(5 + \sqrt{x}) = y\sqrt{x} + 21$

$40 + 8\sqrt{x} - 5\sqrt{x} - x = y\sqrt{x} + 21$

$40 + 3\sqrt{x} - x = y\sqrt{x} + 21$ (1)

Compare like terms :

$40 - x = 21$

$40 - 21 = x$

$x = 19$ (1)

$3\sqrt{x} = y\sqrt{x}$

$y = 3$ (1)

$x =$ 19

$y =$ 3

(Total for Question 5 is 3 marks)

6 Express $\dfrac{3+\sqrt{8}}{\left(\sqrt{2}-1\right)^2}$ in the form $p + \sqrt{q}$ where p and q are integers.

Show each stage of your working clearly.

$$\frac{3+\sqrt{8}}{(\sqrt{2}-1)(\sqrt{2}-1)}$$

$$= \frac{3+\sqrt{8}}{2-2\sqrt{2}+1} \quad (1)$$

$$= \frac{3+\sqrt{8}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} \quad (1)$$

$$= \frac{9+6\sqrt{2}+3\sqrt{8}+2\sqrt{16}}{9-4(2)}$$

$$= \frac{9+6\sqrt{2}+3(2\sqrt{2})+2(4)}{1} \quad (1)$$

$$= 17+12\sqrt{2}$$

$$= 17+\sqrt{12^2\times 2} = 17+\sqrt{288} \quad (1)$$

$17+\sqrt{288}$

(Total for Question 6 is 4 marks)

7 Without using a calculator, show that $\dfrac{12}{\sqrt{2}-1} - \left(\sqrt{2}\right)^5 = 2\sqrt{32} + 12$

Show your working clearly.

$$\frac{12}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{12\sqrt{2}+12}{1} \quad (1)$$

$$12\sqrt{2} + 12 - (\sqrt{2})^5$$

$$12\sqrt{2} + 12 - \sqrt{32} \quad (1)$$

$$12\sqrt{2} + 12 - 4\sqrt{2}$$

$$8\sqrt{2} + 12$$

$$8\sqrt{2} = 2\sqrt{4^2 \times 2} \quad (1)$$

$$= 2\sqrt{32}$$

$$\therefore 2\sqrt{32} + 12 \text{ (shown)}$$

(Total for Question 7 is 3 marks)

8 $a = \sqrt{8} + 4$

$b = \sqrt{8} - 4$

$(a - b)(a + b)$ can be written in the form $y\sqrt{4y}$

Find the value of y
Show your working clearly.

$a - b = \sqrt{8} + 4 - (\sqrt{8} - 4)$

$= 8$ 1

$a + b = \sqrt{8} + 4 + (\sqrt{8} - 4)$

$= 2\sqrt{8}$

$(a-b)(a+b) = 8(2\sqrt{8})$ 1

$= 8(\sqrt{4 \times 8})$

$y = 8$ 1

$y =$ 8

(Total for Question 8 is 3 marks)

9 The diagram shows the prism *ABCDEFGHJK* with horizontal base *AEFG*

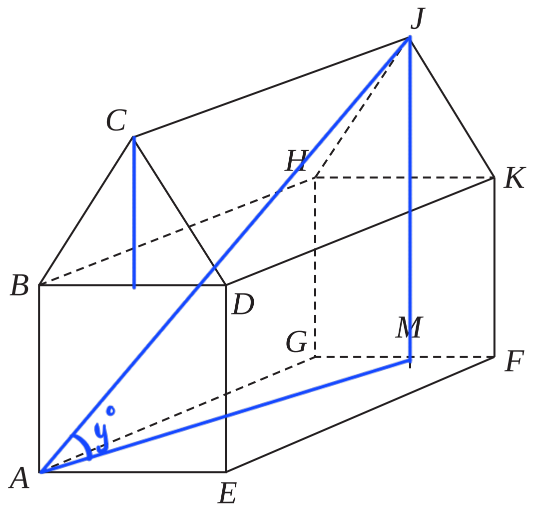

Diagram **NOT** accurately drawn

ABCDE is a cross section of the prism where
ABDE is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of *GF* so that *JM* is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of *T*, giving your answer in the form $\dfrac{\sqrt{p}+\sqrt{q}}{17}$ where *p* and *q* are integers.

let $GM = x$

$GF = 2x$

$EF = 4x$

$AM = \sqrt{x^2 + (4x)^2}$

$= \sqrt{17x^2}$ (1)

$= \sqrt{17}\,x$

Height of triangle $= \sqrt{(2x)^2 - x^2}$

$= \sqrt{3}\,x$ (1)

$JM = 2x + \sqrt{3}\,x$

$$\tan y^\circ = \frac{2x + \sqrt{3}\,x}{\sqrt{17}\,x} \quad (1)$$

$$\tan y^\circ = \frac{2+\sqrt{3}}{\sqrt{17}} = T$$

$$\frac{2+\sqrt{3}}{\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}} \quad (1)$$

$$\frac{2\sqrt{17} + \sqrt{51}}{17}$$

$$= \frac{\sqrt{4\times17} + \sqrt{51}}{17}$$

$$= \frac{\sqrt{68} + \sqrt{51}}{17} \quad (1)$$

$T = \frac{\sqrt{68} + \sqrt{51}}{17}$

(Total for Question 9 is 5 marks)

10 Show that $\dfrac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a + \sqrt{b}$ where a and b are integers.

$$\frac{\sqrt{12}}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} \quad (1)$$

$$= \frac{\sqrt{36} - 2\sqrt{12}}{3-4} \quad (1)$$

$$= \frac{6 - \sqrt{4 \times 12}}{-1}$$

$$= -6 + \sqrt{48} \quad (1)$$

(Total for Question 10 is 3 marks)

11 Solve $\sqrt{3}(x - 2\sqrt{3}) = x + 2\sqrt{3}$

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.
Show your working clearly.

$\sqrt{3}x - 2(3) = x + 2\sqrt{3}$

$\sqrt{3}x - x = 6 + 2\sqrt{3}$ (1)

$x(\sqrt{3} - 1) = 6 + 2\sqrt{3}$

$x = \dfrac{6 + 2\sqrt{3}}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$ (1) (1)

$= \dfrac{6\sqrt{3} + 6 + 2(3) + 2\sqrt{3}}{3 - 1}$

$= \dfrac{12 + 8\sqrt{3}}{2}$

$= 6 + 4\sqrt{3}$ (1)

$x =$ $6 + 4\sqrt{3}$

(Total for Question 11 is 4 marks)

12 Given that $8\sqrt{m} + \sqrt{49m} - \sqrt{9m} = k\sqrt{m}$

where k is an integer and m is a prime number,

(a) work out the value of k

$8\sqrt{m} + 7\sqrt{m} - 3\sqrt{m}$

$= 12\sqrt{m}$ (1)

$k = $ 12

(1)

(b) Show that $\dfrac{5-\sqrt{18}}{1-\sqrt{2}}$ can be written in the form $a + b\sqrt{2}$

where a and b are integers.
Show each stage of your working clearly.

$\dfrac{5-\sqrt{18}}{1-\sqrt{2}} \times \dfrac{1+\sqrt{2}}{1+\sqrt{2}}$ (1)

$= \dfrac{5+5\sqrt{2}-\sqrt{18}-6}{1-2}$

$= \dfrac{5-6+5\sqrt{2}-3\sqrt{2}}{-1}$ (1)

$= 1-2\sqrt{2}$ (1)

(3)

(Total for Question 12 is 4 marks)

13 Show that $\dfrac{2\sqrt{3}}{\sqrt{3}-1}$ can be written in the form $a + \sqrt{a}$ where a is an integer.

Show your working clearly.

$$\frac{2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad (1)$$

$$= \frac{2(3) + 2\sqrt{3}}{3-1}$$

$$= \frac{6+2\sqrt{3}}{2} \quad (1)$$

$$= 3 + \sqrt{3} \quad (1)$$

(Total for Question 13 is 3 marks)

14 The diagram shows a cuboid with a square cross section.

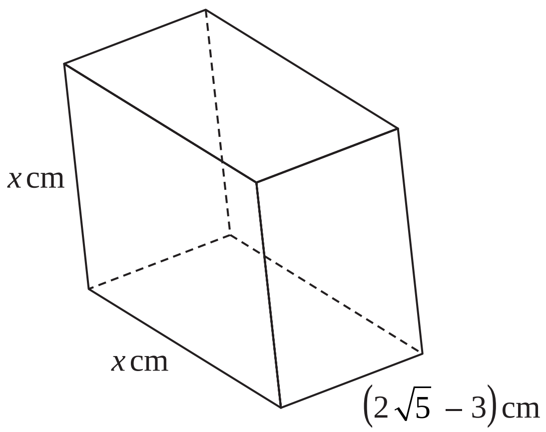

Diagram **NOT** accurately drawn

The volume of the cuboid is $(13+6\sqrt{5})\text{cm}^3$

Without using a calculator, find the value of x
Give your answer in the form $a+\sqrt{b}$ where a and b are integers.
Show your working clearly.

$$x \times x \times (2\sqrt{5}-3) = 13+6\sqrt{5}$$

$$x^2 = \frac{13+6\sqrt{5}}{2\sqrt{5}-3} \quad (1) \quad \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3} \quad (1)$$

$$= \frac{26\sqrt{5}+39+12(5)+18\sqrt{5}}{4(5)-9}$$

$$= \frac{39+60+26\sqrt{5}+18\sqrt{5}}{11}$$

$$= \frac{99+44\sqrt{5}}{11} \quad (1)$$

$$x^2 = 9+4\sqrt{5}$$

$$x = a+\sqrt{b}$$

$$x^2 = a^2+2a\sqrt{b}+b$$

$a^2 + b = 9$

$2a\sqrt{b} = 4\sqrt{5}$

$2a = 4$

$a = 2$, $b = 5$

$x =$ $2 + \sqrt{5}$ (1)

(Total for Question 14 is 4 marks)